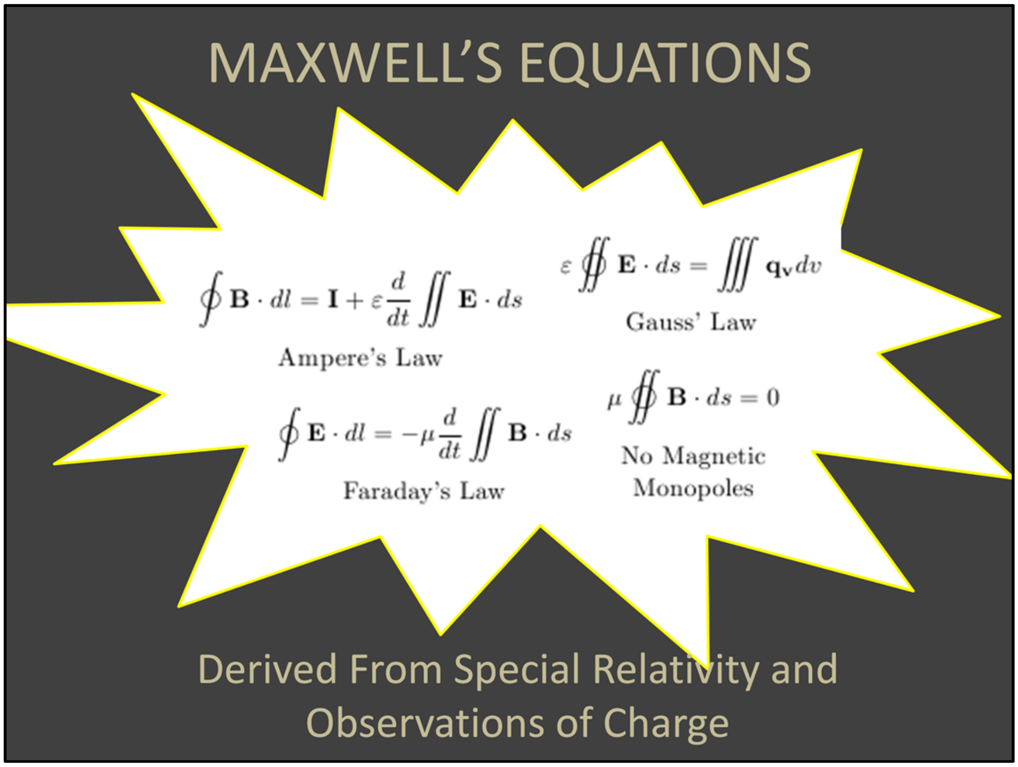

MAXWELL'S EQUATIONS
$\oint \mathbf{B} \cdot dl = \mathbf{I} + \varepsilon \frac{d}{dt} \iint \mathbf{E} \cdot ds$
Ampere's Law
$\varepsilon \oiint \mathbf{E} \cdot ds = \iiint \mathbf{q_v} dv$
Gauss' Law
$\oint \mathbf{E} \cdot dl = -\mu \frac{d}{dt} \iint \mathbf{B} \cdot ds$
Faraday's Law
$\mu \oiint \mathbf{B} \cdot ds = 0$
No Magnetic Monopoles
Derived From Special Relativity and Observations of Charge

WELCOME!

# Summary of Approach

As I mentioned in the description, this class is going to very quickly cover the core of 2 college level math classes and 1 physics class

Breakdown:

- The Variables of the equations – What are we talking about
  - Characterizing Charge and the Electric Field
- The Mathematical Relations of the equations
  - Single and Multivariable Calculus in a nutshell
- Gauss's Law
- Special Relativity – where intuition Breaks Down
  - Deriving the Magnetic Field
- Describing the Magnetic Field with The Fourth Equation
- Describing the Magnetic Field with Ampere's Law and Faraday's Law
- Applying the Mathematics to understand Dynamo Theory (the physics of the earth's magnetic field)

# Before We Start

NAMES:
- Zandra Vinegar,
- I'm a teacher at MoMath
- I graduated from MIT last year (18 – math)

QUESTIONS:
- After class
- Email: ch3cooh@mit.edu
- Website: https://sites.google.com/site/applelessgarden/vinegar

DON'T TAKE NOTES:
- Don't try to take notes
- Printouts
- posting it online.

GETTING LOST:
- You probably will
- Follow the themes
- Fill in the details at home

Don't sweat the details, but don't let your mind wander. I've put break slides throughout this presentation so that I remember to give you breathers, so stick with me, they should come around about every ½ hr or so, starting – Now

ASK THE STUDENTS THEIR NAMES: I always forget to ask my students their names. This slide should remind me to ask your names (quickly). Oh, and my name is Zandra Vinegar, I'm a graduate of MIT and a current employee at MoMath. ☺ The museum of mathematics in Manhattan NY

DON'T TAKE NOTES: Don't try to take notes unless you can do it without thinking. I've got printouts of this presentation for each of you, and I'm posting it online.
QUESTIONS: Usually, I love to get off topic in my classes and dive into whatever students seem interested in. In this class, we have *2hrs* to cover about 20 hrs of mit course material. If you want me to clarify a statement, or if you catch a mistake that you think I've made, raise your hand…etc. I'll stay after class and my email is ch3cooh@mit.edu. If you're writing that down, this is a bad sign, see built #2.

GETTING LOST: You probably will, at some point, hopefully not within the first hour, but hold on and try to follow the themes of the class – how the ideas connect, and, with the help of the internet and this presentation, at home you can fill in the details.

Don't sweat the details, but don't let your mind wander. I've put break slides throughout this presentation so that I remember to give you breathers, so stick with me, they should come around about every ½ hr or so, starting - Now

# Defining Variables

- B – Magnetic Field
- E – Electric Field
- q – Charge Density
- I – Current (moving charge)
- l – loop
- s – surface
- v – volume

Constants We Won't Deal With
(measured, not calculated)
("is proportional to") – makes the speed
Of light = c and not 5m/s or anything else

$\varepsilon$ permittivity of free space,
officially the electric constant

$\mu$ permeability of free space,
officially the magnetic constant

$$\oint \mathbf{E} \cdot dl = -\mu \frac{d}{dt} \iint \mathbf{B} \cdot ds$$

Faraday's Law

$$\varepsilon \oiint \mathbf{E} \cdot ds = \iiint \mathbf{q_v} \, dv$$

Gauss' Law

$$\oint \mathbf{B} \cdot dl = \mathbf{I} + \varepsilon \frac{d}{dt} \iint \mathbf{E} \cdot ds$$

Ampere's Law

$$\mu \oiint \mathbf{B} \cdot ds = 0$$

No Magnetic Monopoles

In SI units the speed of light in vacuum $c_0$ is defined[3] as the numerical value $c_0$ = 299 792 458 m $s^{-1}$ (See $c_0$) and the magnetic constant $\mu_0$ is defined as $4\pi \times 10^{-7}$ H · $m^{-1}$ (See $\mu_0$), leading to the definition for the electric constant of $\varepsilon_0 = 1/(\mu_0 c_0^2) \approx 8.854\,187\,817 \times 10^{-12}$ F $m^{-1}$

# Charge, Q, and The Electric Field, E

- Charge is an observed property of matter
  - Negative repels negative
  - Positive repels positive
  - Negative attracts positive, and the reverse
  - There are strongly charged particles
  - And weakly charged particles
  - The force between two particles decreases rapidly with distance
- BUT CHARGES ARE NOT MAGNETS, THEY ARE DIFFERENT
- Electric Force = $\frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}$ = $kq_1q_2/r^2$
  - In SI units
- Note: Force is a Vector – it has a magnitude and a direction.

This entire class is about Deriving what The Magnetic Field, H is, but we have to start by understanding what we mean by charge and electric fields

We observe that particles of different types exert amounts of force on each other. “negatively charged particles” which, independent of their other properties, seem to repel other negative particles and attract “positively repelling particles” Also, independent of other properties, these particles seem to be able to repel or attract with different amounts of force: we observe *strongly* negative and *weakly* negative charges, which correspondingly, have significant or slight effect on surrounding charged particles. Additionally, we observe that the force of one charge on another is dependent on distance such that, as two particles are separated by greater distances, they have less effect on each other.

Specifically, we characterize the entire force between two charges as $kq_1q_2/r^2$
Where k is a constant dependent on the units of charge and force being used.

Example:

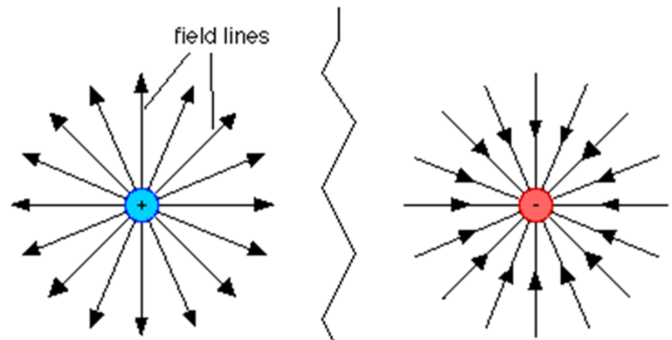
# Electric **FIELD** of a point charge

The electric field of a charge is a function over 3D space which describes the strength of the point charge at an arbitrary point in space. Thus The Electric Field of a single point charge is simply:

$$\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} = \frac{k\,q}{r^2}$$

Note:
**Force is a Vector**
- *magnitude*
- *direction*

The Electric Field is a **Vector Field** with a vector defined for every point in space




The electric field from an isolated positive charge

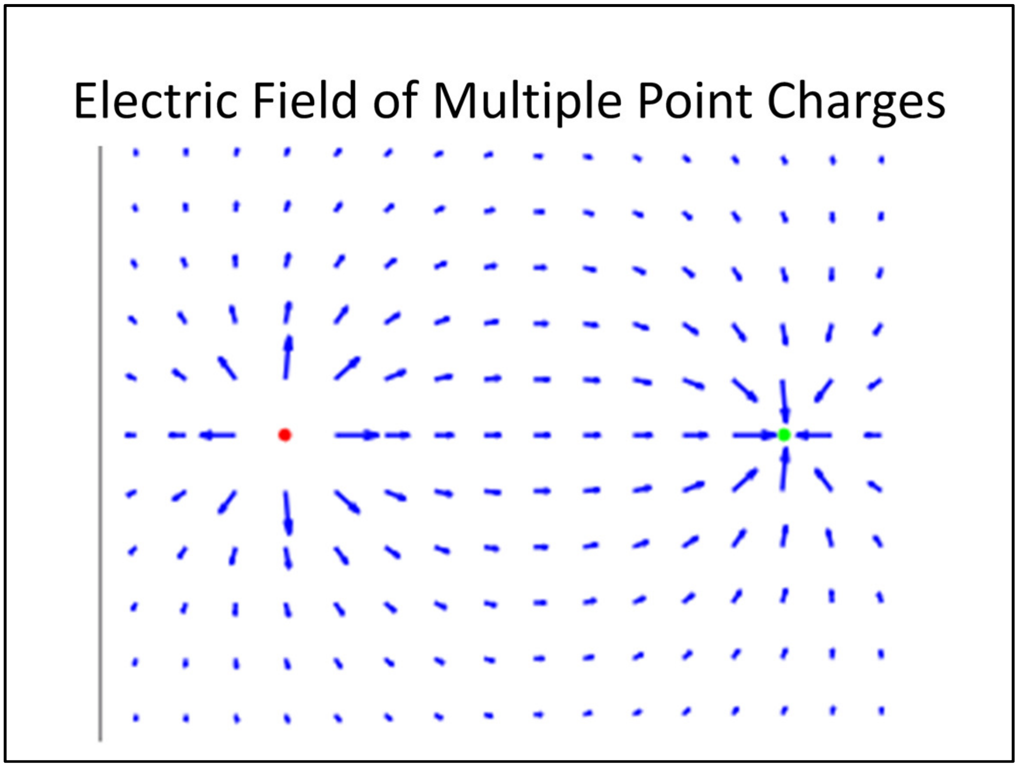
The electric field from an isolated negative charge

The electric field of a charge is a function over 3D space which describes the strength of the point charge at an arbitrary point in space.

This doesn't look like a field over all of space (call the dimensions x, y, z), but think about it, it wouldn't look too pretty in cartesian coordinates, but it is well defined.

Note: Force is a Vector – it has a magnitude and a direction.
This makes Electric fields, "Vector Fields" – an equation which describes a vector at every point in space. The magnitude of the vector describes the strength of the charge's force at that point, and the direction describes the direction in which the force of the charge will take effect.

You can think of a field as: "what would happen if I put a test charge here" for every point in space.



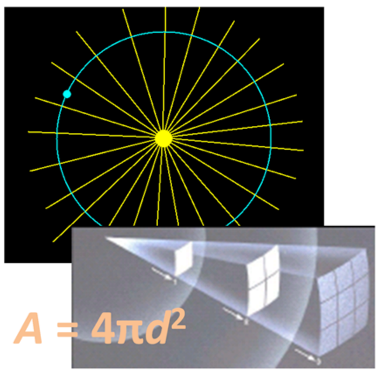

The Electric field of a bunch of point charges is just the sum of the fields of the individuals (keeping track of translation)

# “Chargons (virtual photons)”

Point charges “radiate ‘chargon’ particles (NOT an official term in physics – the official term is **virtual photons**)”

Thus, as you get further away from the charge – the source, the particles are more spread out and fewer of them effect any given other charge.

$$\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} = \frac{k\,q}{r^2}$$

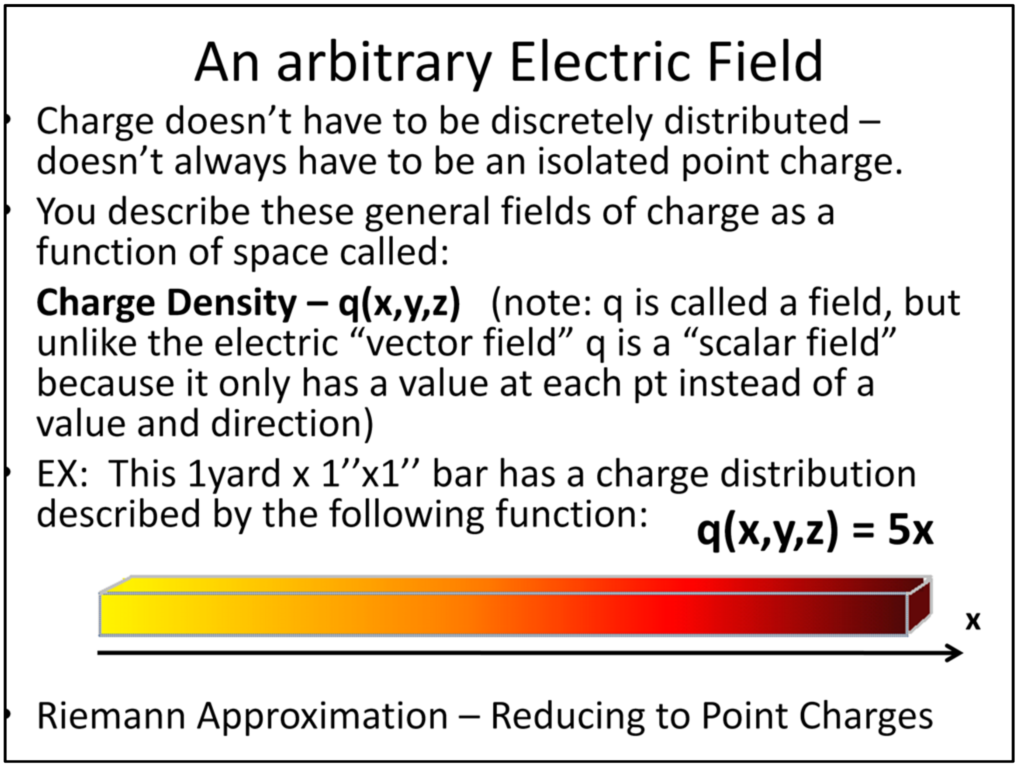



***So p particles are spread out over 4πr$^2$ space so their effect will be proportional to p/r$^2$ .   If p is proportional to Q, the strength of a charge, then E, the strength of the electric field is proportional to p/r$^2$***

One way to think about this relation is that, given the strength of a point charge, every second it radiates out so many equal strength “chargon” particles (NOT an official term in physics)  Thus, as you get further away from the charge – the source, the particles are more spread out and fewer of them effect any given other charge.

Like the picture, Except that circle should be a sphere, since the particles will be sent out through all of 3D space.  Therefore, a distance r away, 1 second’s worth of “chargons” are spread out over the surface are of a sphere with radius r.


GIVE CLASS
BREAK




Charge doesn't have to be discretely distributed – doesn't always have to be an isolated point charge.

You can have, for instance a charged balloon that has charge uniformly distributed across its surface. Or a charged bar that has a slight charge on one end which varies continuously until it is very charged on the other end.
You describe these general fields of charge as a function of space called:

**Charge Density – q(l,w,h)** (note: q is called a field, but unlike the electric "vector field" q is a "scaler field" because it only has a value at each pt instead of a value and direction)
For point charges, recognizing q is simple. q = Q1 at point (a, b, c) and 0 everywhere else. Calculating the electric field is simply a matter of superimposing the electric fields of each point charge described by Q1/r
EX: This yard long 1''x1'' bar has a charge distribution described by the following function. … now it's not quite as obvious how to calculate the electric field. An arbitrarily close approximation is easy, just treat every 1''cube of the bar as a point charge at the center with charge = the area of the cube times the average density of charge across the cube. For a better approximation, you can do this with every ½''cube of the bar. Etc.
Reducing to point charges

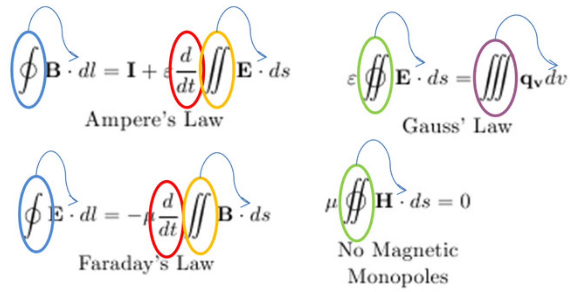
# Variables for Gauss's Law – Check ☺

$$\varepsilon \oint\!\!\oint E \cdot ds = \iiint q_v dv$$

- Gauss's Law only refers to charge and the electric field, summarizing the properties we just described
- Now that you know what the variables mean, lets talk about the Mathematical notation, which simply expresses, in mathematics, the description of charge and electricity you just learned

# Mathematical Notation

$\int$ – An "Integral"




Derivative – NOT DIVISION – (/dt – with respect to time)

Integral over a closed loop – taken wrt a loop

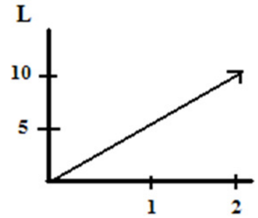
Double Integral – taken wrt a surface

Double Integral over a "Closed Surface" – wrt a surface

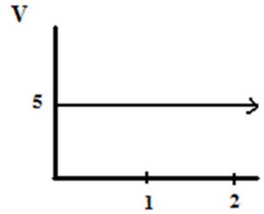
Triple Integral – wrt a volume (cool note, to take the next step, a "closed volume" would need to be realized in 4D space ☺

# What is a Derivative

Intuitively: the rate at which a function changes




$\frac{\mathbf{d}}{\mathbf{dt}} \mathbf{L} = \mathbf{V}$




$\frac{\mathbf{d}}{\mathbf{dt}} \mathbf{V} = \mathbf{A}$

SLOPE

$$m = \frac{\Delta f(x)}{\Delta x} = \frac{f(a+h) - f(a)}{h}.$$

FORMAL DEFINITION OF THE DERIVATIVE

$$\frac{\mathbf{d}}{\mathbf{dt}} f\ (a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

Ex: Velocity is the Derivative of Motion. Acceleration is the Derivative of Velocity. Jerk is the Derivative of Acceleration.

d/dt , the derivative of a function with respect to t, time, is the rate of change of the function (can also vary with time)

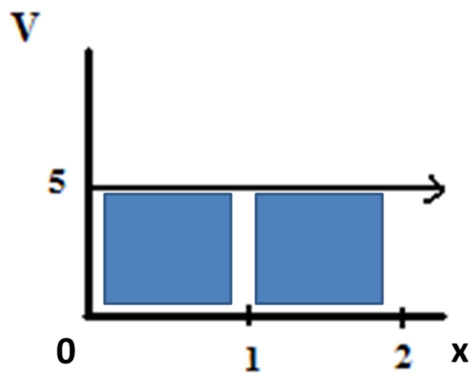
For example. Consider the function of an ant who's location on a line can be described as L=5t meters. The rate of chage of the location is therefore 5 meters/second. Thus d/dt of L is 5.

Alternatively, the ant's velocity could vary with time, thus, d/dt doesn't have to be a number, it can be a function that also varies with time.

# What is an Integral

If the **derivative** of Y(t) is X(t)

the **integral** of X(t) is Y(t)




What, on this graph is changing at a rate described by the function, V?

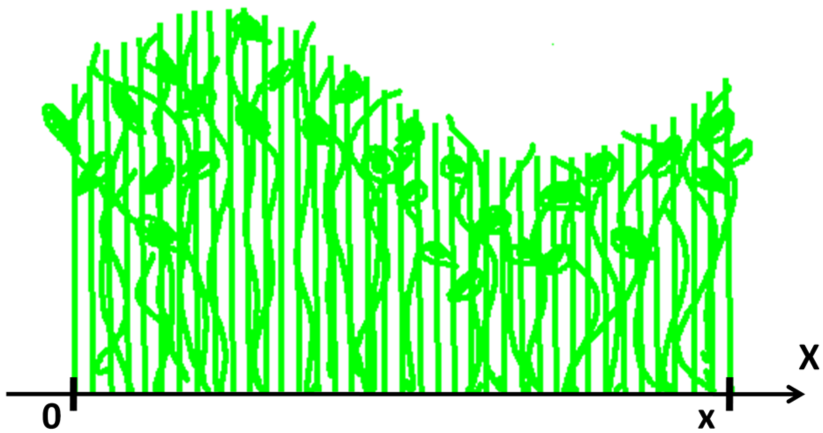
**INTEGRAL = THE AREA UNDER A CURVE**

The integral is pretty much just the reverse operation of the derivative. However, you should notice that, since a lot of different functions have the same derivative, the integral of a function is actually a family of closely related functions. For example, the lines y=5, y=39, and y=pi all have derivative 0. Therefore, the integral of 0 is any one of those constants, or any constant for that matter, C.

See if you can figure out what the family of integrals have in common for an arbitrary function. The uniqueness theorem of Calculus, basically, that families of integrals don't overlap, should now be fairly intuitive to you. If not, try to make a counterexample – a function which has two different derivatives. Just thinking about the integral as described above should make it clear that this is impossible.

# Integrals over SPACE (lines, surfaces, and volumes):

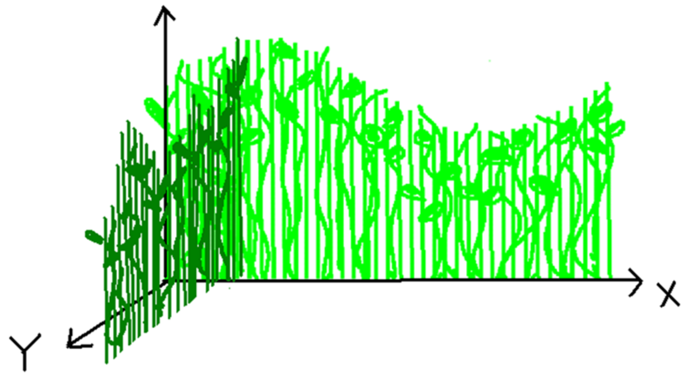
- Say you have a row of bean stalks and you know the height of this row as a function of x,
- B(x)




- The amount of bean plant you have is the integral of B (the integral over the line from 0 to x)

# Multi-variable Integral

- But what if you have a field of bean stalks with height known as a function of x and y, F(x,y)

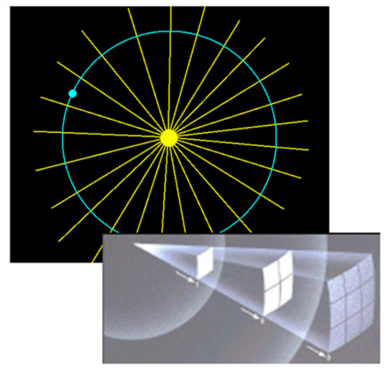



1. First, allowing x to vary, take the integral of F(x,y) with respect to y – Aka, for x = 5 integrate F(5,y) with respect to y, this finds the total amount of bean plant in the sheet at x=5. Doing this for x results in a function, T(x), which is the total amount of stalk at each x location. This is the "Projection of y onto the x axis. You can see that the function F(x,y) describes a surface – the tops of all the corn stalks. The double integral is therefore just the amount of corn under the surface.

## Gauss's Law: $\varepsilon \oiint E \cdot ds = \iiint q_v dv$

- Right Half – the total amount of charge within some volume of space, v
- This volume is enclosed by a surface, s
- Left Half – Electric field over the entire surface

Means exactly what we described earlier – that, because of the nature of the force of charge, you could think about charges as **sources** of "chargons" which radiate out from the charges. Therefore, with any surface, you expect to catch **the same** total number of "chargons" proportional to the amount of charge within the surface



Gauss's law is simply the mathematical method for ensuring what we described earlier as the property of charge as a "chargon" source => creating the electric field.

The Right Half

So charge is a scalar field q(x,y,z) therefore, the tripple integral of q over a volume, v, is the total amount of charge within the volume.
Consider BY TREND
– single integral is the area under a curve = the total amount of something that has values at every point on a line.
-Double integral is the volume under surface of something that has a value at every point on a plane
- q has a value at every point in a 3 dimensional space. Therefore the triple integral of q over the volume, v, is the total amount of charge within the volume.
BY PROJECTION
dv = dz dx dy. So the inner most integral finds the total amount of charge at all heights for a point on the x,y plane = r(x,y). The next integral finds the total amount of charge for a point on x, an integral dy. = s(x) The last integral finds the total amount of charge in the entire volume, Q = the integral of s(x) dx.

V is a volume in space – therefore it's border is a "closed surface" a surface with no edges – like a balloon surrounding air. Recall that the electric field is like the number of chargons that reach a part of space every second. Therefore, the left half of the equation is how many chargons Total are at the surface – the double integral over the entire surface.

This describes exactly the behavior of charge that we talked about earlier – that the electric field created by a point charge is $kq/r^2$

# The Fourth Equation

- We're not ready for the fourth equation yet, because I haven't explained *anything* about the magnetic field, but, from a purely mathematical standpoint, you can figure out exactly what this law means – exactly what it describes about the magnetic field

$$\varepsilon \oiint \mathbf{E} \cdot ds = \iiint \mathbf{q_v} dv \qquad \mu \oiint \mathbf{B} \cdot ds = 0$$

Gauss' Law　　　No Magnetic Monopoles

**THERE ARE NO SOURCES OF MAGNETICIC FIELDS – "MAGNATRONS" ARE CONSERVED, LIKE WATER FLOWING THROUGH SPACE**

now, to see *why* this is true.

THERE ARE NO SOURCES OF MAGNETICIC FIELDS – "MAGNATRONS" ARE CONSERVED, LIKE WATER FLOWING THROUGH SPACE


GIVE CLASS
BREAK

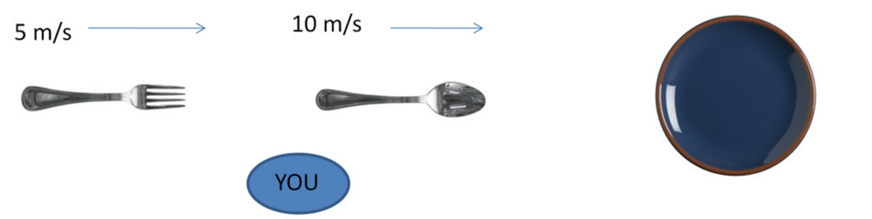
## SPECIAL RELATIVITY – Deriving The Magnetic Field

# Special Relativity

Inertial Observers – observers moving at any constant speed. (acceleration gets messy -> general relativity)

How do Inertial objects behave:

1. Relative Motion
2. Addition of Relative Velocities



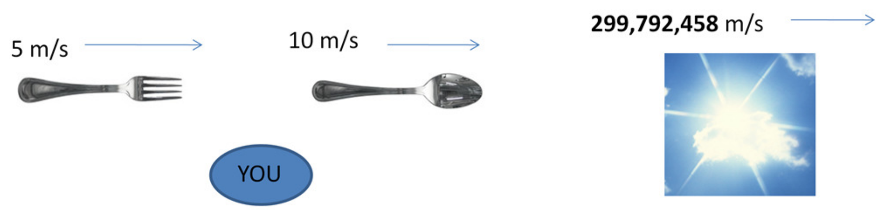
# Addition of Relative Velocities is FALSE

We always measure light to be moving at

**299,792,458 m/ s = c**

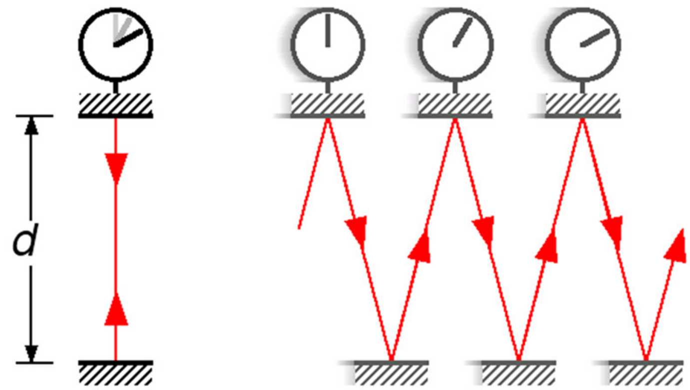
This means that if a fork and a spoon are running after a moon beam, even if the spoon is running and the fork is walking, they both measure that the moonbeam escapes away from them at a relative speed of c.

THIS BREAKS YOUR INTUITIVE, CLASSICAL, SENSE OF THE UNIVERSE IN A NUMBER OF WAYS



# Time Dilation

c = speed = distance/time

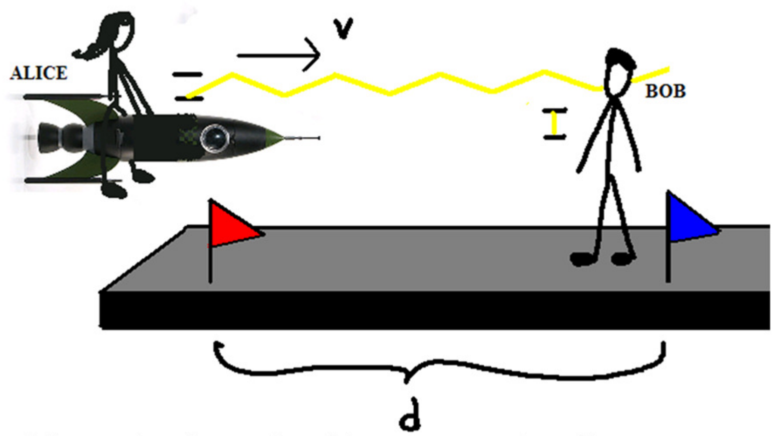



shorter distance
-> time runs slower

longer distance
-> time runs faster

# Lorenz Contraction

- Measure time in Light clock ticks



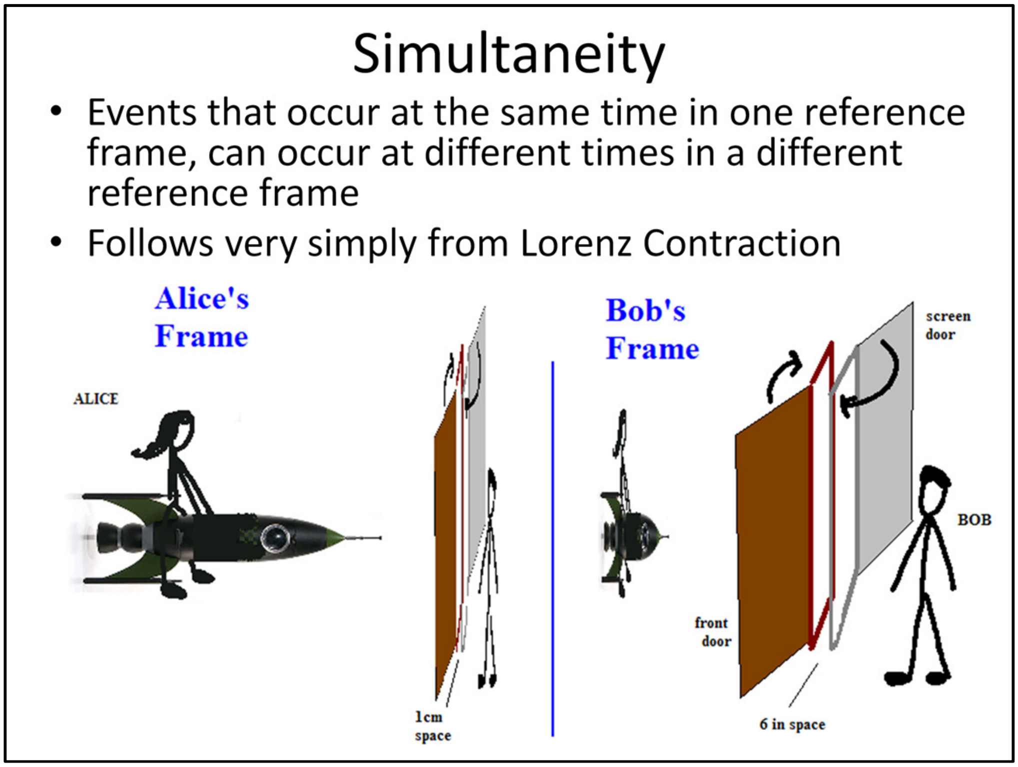

- To Alice, the length, d between the flags appears shorter – objects moving past quickly contract.
  (therefore, to Bob, Alice's ship is shorter than it is to Alice)

In Bob's reference frame, Bob sees Alice pass at a velocity, v. In Alice's reference frame, Alice sees the ground pass with velocity v.

If Bob and Alice both have light clocks, and they both start counting ticks when Alice reaches the red flag and stop counting ticks when Alice reaches the blue flag, what do Alice and Bob conclude about d?

Clearly Alice counts fewer ticks than Bob because Bob has the length of time that it takes Alice's clock to make that long path, to run his clock simply vertically.
Therefore, Bob thinks that Alice traveled for longer than Alice thinks she did. Since they agree on a relative velocity of v, and that Alice traveled a distance d, Bob must think that d is longer than Alice thinks it is. In other words, to Alice, the ground contracts and the flags are closer together. To Bob, the length of Alice's ship is shorter than it seems to Alice = LENGTH CONTRACTION

DO THE MATH!!! – In fact, contraction is proportional to velocity:
Math:




Bob just learned about relativity and wants to have some fun with it. He realizes that "Alice, if you can go fast enough, I can trap your rocket for one instant between my front door and the inner screen door – six inches apart. You just go fast enough that the length of your rocket contracts to 6inches long, and then, for the instant that you are between the doors, I'll shut both of them, and then immediately open them so that you can fly through" Alice thinks this is ridiculous, but, for some reason goes along with it any way. Except, as she is flying towards the door, she is suddenly struck by a panic attack when, despite her knowledge of physics which makes this predictable, she sees that, to her, the space between the doors is now only a centimeter wide – it contracted in her reference frame. She is never going to fit. But, as she approaches the door, just as her front reaches the screen door, Bob closes the screen door for one instant. But opens it so that she can pass through the door. Then, just her tail reaches the front door, Bob closes the front door for an instant. Alice relaxes, she won't tell Bob how silly he looked shutting a door infront of her, but when she radios in, Bob declares the experiment a success!

Lesson Learned: funny things happen when you go at relativistic speeds.

Pretty simple, no? And we only really need Lorenz Contraction for deriving Magnetism.

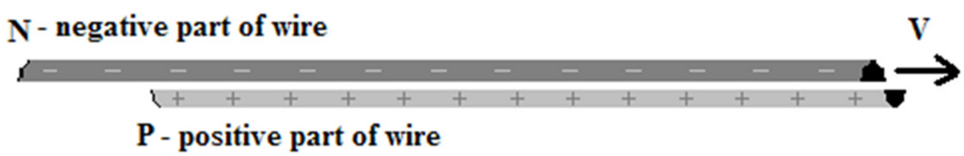
# A Current in An INFINITE Wire

- Wires – made of atoms
  - equal numbers of protons and electrons
- Simplification (infinite wire):

- Current – a flow of the electrons in the wire

IN THE REFERENCE FRAME OF THE POSITIVE CHARGES the negative charges move

- Modification in light of special relativity:
  - start with a less dense (wrt charge) negative bar (STRETCH IT)
  - equal charge density in the reference frame of the positive charges




Wires – made of atoms (not charged overall therefore there must be equal numbers of protons and electrons – positively and negatively charged particles)
Simplification:

Current => a flow of the electrons in the wire => negative half of the wire moves. However, taking relativity into account, moving this bar makes it contract relative to the positive bar. So we start with a less dense (wrt charge) negative bar, then move it at a negative velocity to create a current, but have equal charge density in the reference frame of the positive charges

# A Current in a MOVING infinite wire

**REFERENCE FRAME OF THE POSITIVE WIRE**


Now consider the reference frame of N, the negative wire, moving right at 3/4c

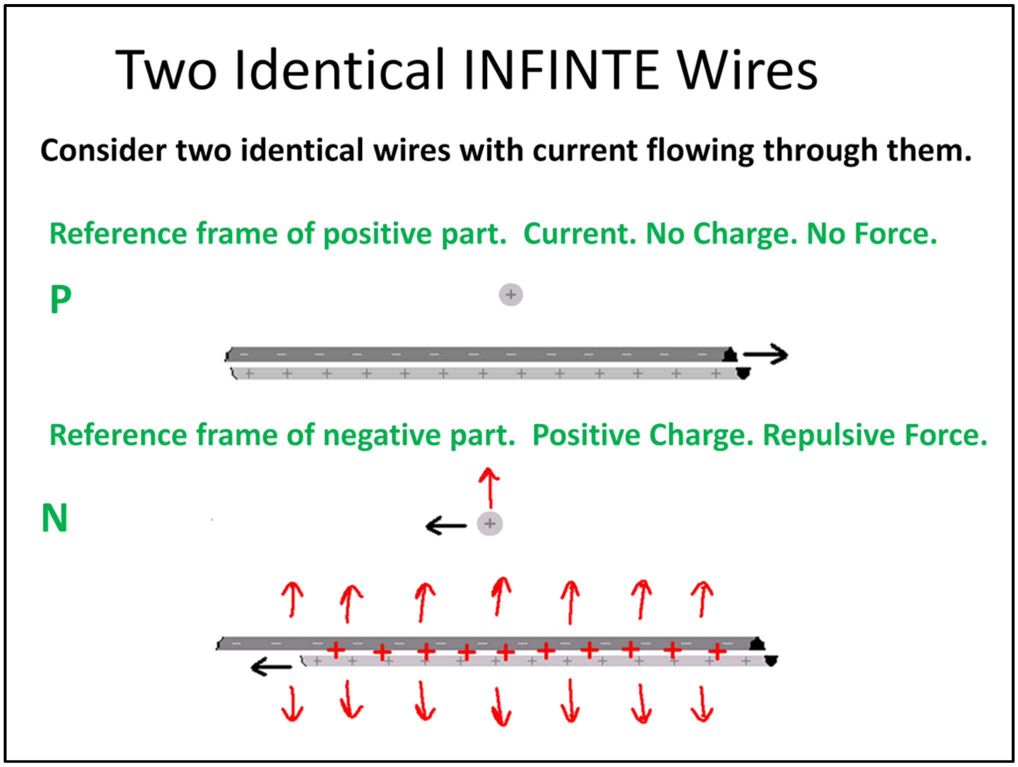
Thus, the negative part of the wire is in its extended state, with a **low charge density**.

Alternately, the positive wire is contracted in the reference frame of N. Thus, there is **very dense positive charge**

NOOOOOOOOO! We failed! – the wire is positively charged in some reference frame! Why is this a problem?

In the Reference frame of the positive wire, let v be measured as ¾c. ¾ of the speed of light. But the negative part of the wire was initially calibrated such that, in the reference frame of the positive wire, the charge desnsities are equal, although Now consider the reference fr

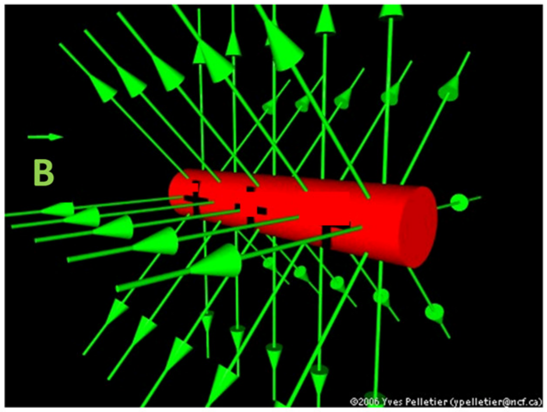
NOOOOO!!! We failed, when you move along the wire – the wire becomes charged! Why is this a problem?….




Consider a wires with current flowing through it and a positively charged particle nearby. In one reference frame, the reference frame of the positive part of the wire, we can make it so that the wires are neutral, and therefore don't exert any forces on each other. But then, in the negative part's reference frame, the wire is positive and will repel the positive particle. So the positive particle is both repelled and not repelled. This is a contradiction and means that Electricity and Relativity alone contradict each other. In order for both to be true, in the situations where creation of a charge would cause two objects with one charge in one reference frame to act differently in another, another force, one which we term magnetism, must come into play so that the two frames agree on measurements and observations. In other words, magnetism must explain the repelling force on the particle in the reference frame of the natural wire with current, the positive reference frame.

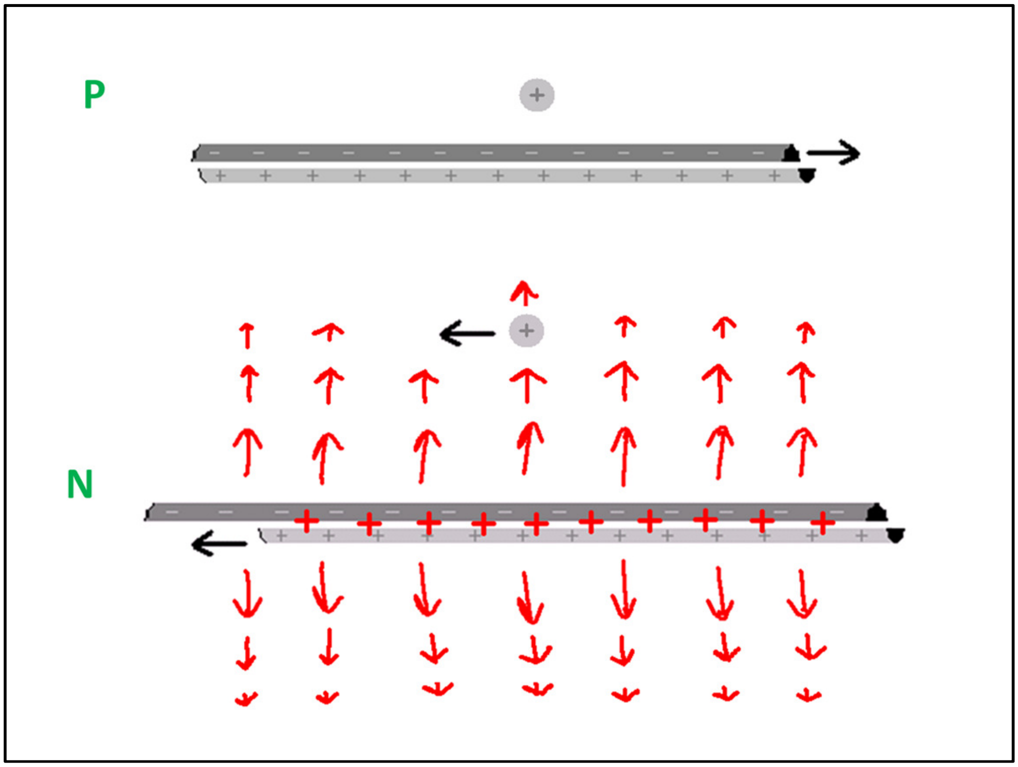
THEREFORE, THE MAGNETIC FIELD EXISTS and is the FORCE OF A MOVING CHARGE ON ANOTHER CHARGE, where this force is proportional to the strength of the charge and how fast the charge is relatively moving and inversely proportional to distance

# Since I can't draw in 3D

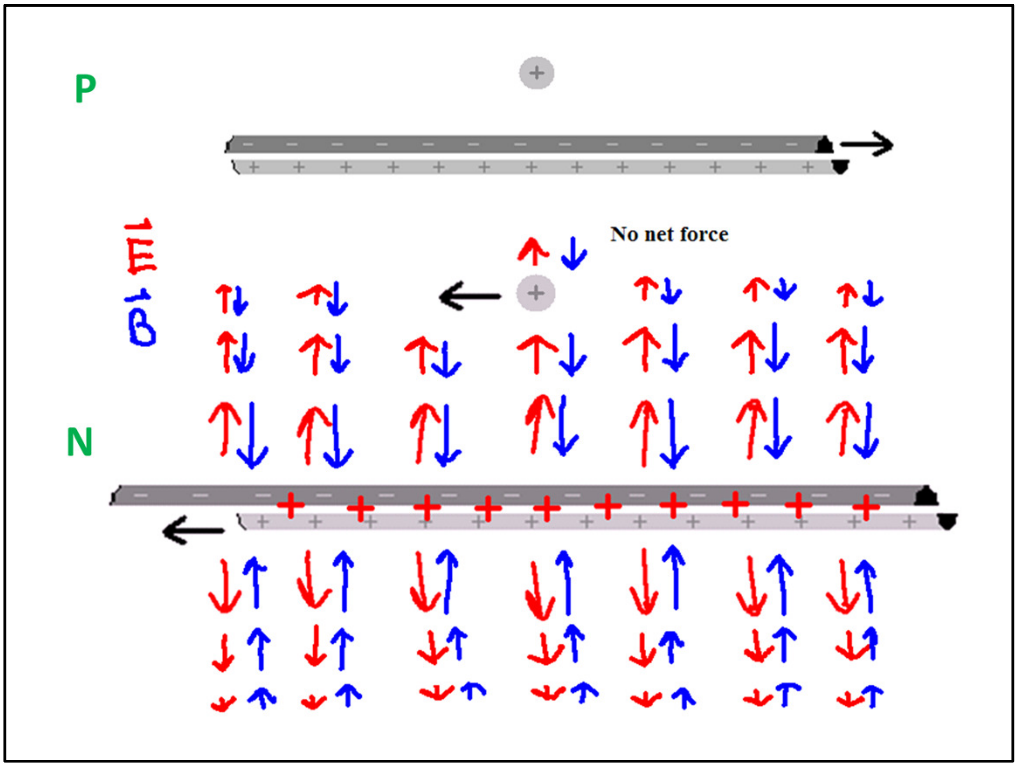



Since I can'


GIVE CLASS
BREAK




THEREFORE, THE MAGNETIC FIELD EXISTS and is the FORCE OF A MOVING CHARGE ON ANOTHER CHARGE, where this force is proportional to the strength of the charge and how fast the charge is relatively moving and inversely proportional to distance


P
No net force
E
B
N

# Technically, now, we're done

The model that we have discussed so far well defines exactly what the electric and magnetic field should be IN ALL PLACES IN SPACE AT ALL TIMES. Now we *just* need to show how the model's description is completed succinctly using the additional Maxwell's equations.

$$\oint \mathbf{B} \cdot dl = \mathbf{I} + \varepsilon \frac{d}{dt} \iint \mathbf{E} \cdot ds \qquad \oint \mathbf{E} \cdot dl = -\mu \frac{d}{dt} \iint \mathbf{B} \cdot ds$$

Ampere's Law Faraday's Law

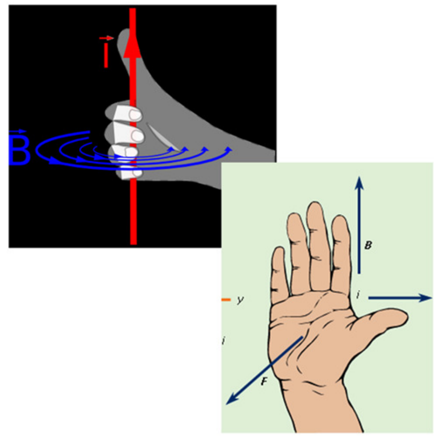
# BUT WAIT! -- Trickiness ☹

Apologies:

#1 Ben Franklin guessed wrong:


WE WERE GOING TO USE THE TIME MACHINE TO PREVENT THE ROBOT APOCALYPSE, BUT THE GUY WHO BUILT IT WAS AN ELECTRICAL ENGINEER.

#2 Physicists decided to recognize the importance of human anatomical form when describing the universe



$$\oint \mathbf{B} \cdot dl = \mathbf{I} + \varepsilon \frac{d}{dt} \iint \mathbf{E} \cdot ds$$

Current

Time Derivative of the Flux of Chargons through the surface, s

A note about Current:

*Positive Current* is defined as the *reverse* of the direction the electrons are moving ☺

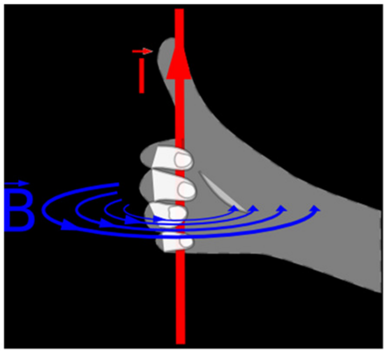

← $I$

Negative charges are moving right, so I is to the left

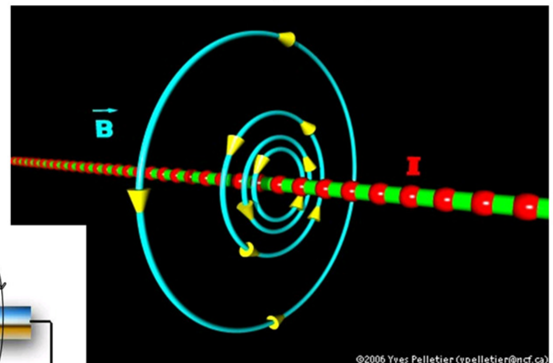
First of all, notice that we need no corrective term for magnetism in all cases of static charge – lorenz contraction has no effect and there is no contradiction. In the Equations, this is reflected by H simply being a sum of the Current (moving charge) and the time derivative of the flux of the magnetron field (no movement, no change with respect to time)

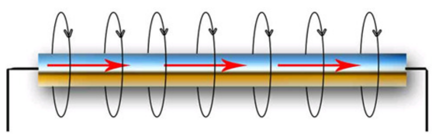
# The Magnetic Field Of a Wire with current flowing through it

**RHR #1**



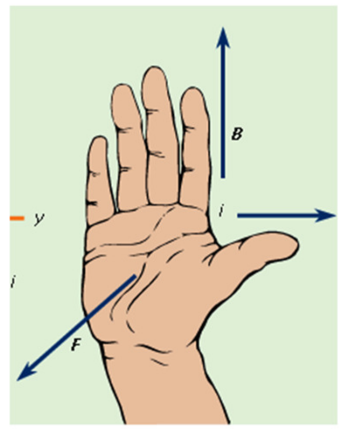

"MATHEMATICAL SIMPLIFICATION" (keeping track of more information in the field description)



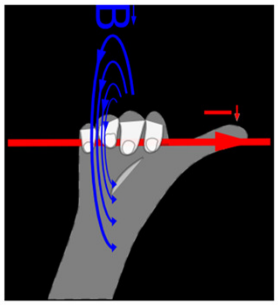
# Right Hand Rule #2

Now, to Undo RHR#1, RHR#2 REDEFINES WHAT A MAGNETIC FIELD LINE IS – it is no longer a force line, but rather, a direction used to find the direction of magnetic force using RHR#2:




If you open you hand and put Your fingers in the direction of The new B and your thumb in The direction of a charged Particle moving in this frame, Then the force will be in the Direction of your palm.

# RHR2 and RHR1 Cancel



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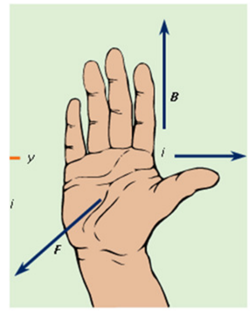


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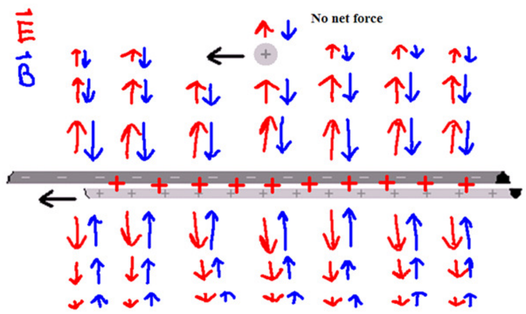


# Describing it with Maxwell's Equations

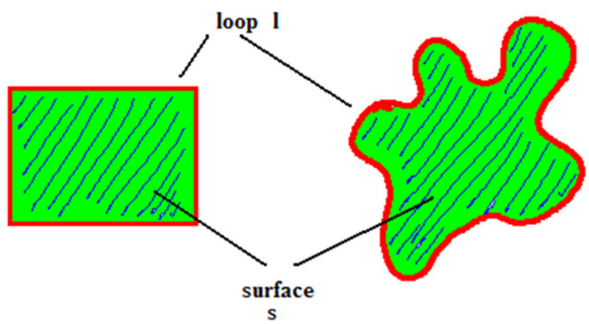
$$\oint \mathbf{E} \cdot dl = -\mu \frac{d}{dt} \iint \mathbf{B} \cdot ds$$

Faraday's Law

$$\oint \mathbf{B} \cdot dl = \mathbf{I} + \varepsilon \frac{d}{dt} \iint \mathbf{E} \cdot ds$$

Ampere's Law

Recall that d/dt is the Derivative: the rate of change of the following quantity with respect to time



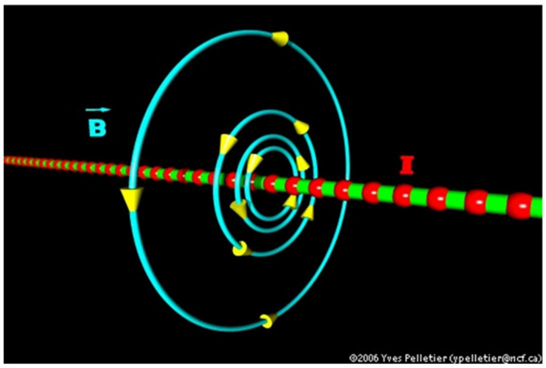

Note that the laws are almost SYMMETRIC: the basic idea is that , the existence of CHANGE in the electric field will define where the magnetic field exists and vice-versa == BECAUSE CHANGE IN A FIELD IMPLIES MOVEMENT WHICH IS WHERE THE SPECIAL RELATIVITY STUFF KICKS IN

FLUX = the rate of flow of chargons or magnatrons through the surface.
EX: if the vector field described the direction and rate of flow of water at every point in space in a river, the flux through a loop lowered into the river would be the gallons/second of water through the loop

$$\oint \mathbf{B}\cdot dl = \mathbf{I} + \cancel{\varepsilon \frac{d}{dt}\iint \mathbf{E}\cdot ds}$$

Ampere's Law

Lets look at our wire system:



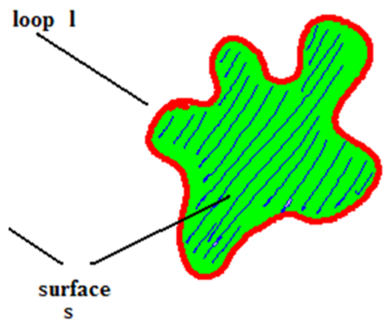

In this case, since, although charges are moving, they are immediately replacing the positions of other charges, the time derivative of E is 0, thus…

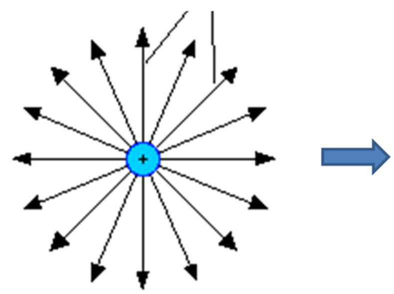
And now it's clear that Ampere's describes exactly what we need according to special relativity: a magnetic field around ANY loop around the wire proportional to the speed of the current. ☺

$$\oint \mathbf{B} \cdot dl = \mathbf{I} + \varepsilon \frac{d}{dt} \iint \mathbf{E} \cdot ds$$

Ampere's Law

The time derivative kicks in when the electric field is changing indicating net movement of charges: How do you tell when a charge is moving? Set up a surface that catches field lines and measure them spreading out as the particle moves! IT MATCHES RELATIVITY!!!



## Why is there no I-like term in Faraday's Law?

A: without magnetic monopoles, there's correspondingly, no sense of a "magnetic current" So the term doesn't need to exist.

$$\oint \mathbf{B} \cdot dl = \mathbf{I} + \varepsilon \frac{d}{dt} \iint \mathbf{E} \cdot ds$$

Ampere's Law

$$\oint \mathbf{E} \cdot dl = -\mu \frac{d}{dt} \iint \mathbf{B} \cdot ds$$

Faraday's Law

Otherwise, Faraday's Law will act just like Ampere's law for instances when there's a changing magnetic field: like a current increasing or decreasing in a wire = Electrical Induction (but that's another class)


WE'RE
DONE!!!!

# Other Forms of Maxwell's Equations

- Of the Equations (from wikipedia)

| Name | Differential form | Integral form |
| --- | --- | --- |
| Gauss's law: | $\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$ | $\oint_S \mathbf{E} \cdot \mathrm{d}\mathbf{A} = \frac{Q_S}{\varepsilon_0}$ |
| Gauss's law for magnetism: | $\nabla \cdot \mathbf{B} = 0$ | $\oint_S \mathbf{B} \cdot \mathrm{d}\mathbf{A} = 0$ |
| Maxwell-Faraday equation (Faraday's law of induction): | $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$ | $\oint_{\partial S} \mathbf{E} \cdot \mathrm{d}\mathbf{l} = -\frac{\partial \Phi_{B,S}}{\partial t}$ |
| Ampère's circuital law (with Maxwell's correction): | $\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$ | $\oint_{\partial S} \mathbf{B} \cdot \mathrm{d}\mathbf{l} = \mu_0 I_S + \mu_0 \varepsilon_0 \frac{\partial \Phi_{E,S}}{\partial t}$ |

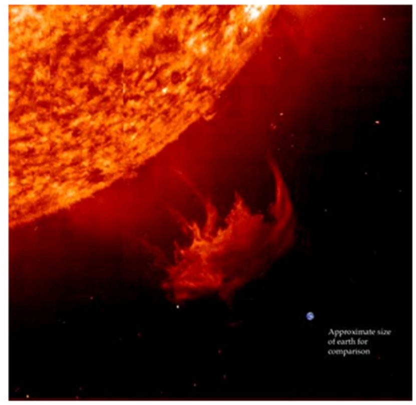
- Of the units:
  - SI (International System of Units)
  - CGS (Centimetre gram second)
  - Lorentz–Heaviside

# Application/Experiment for Home:

## Keeping the earth from going 'foom.'

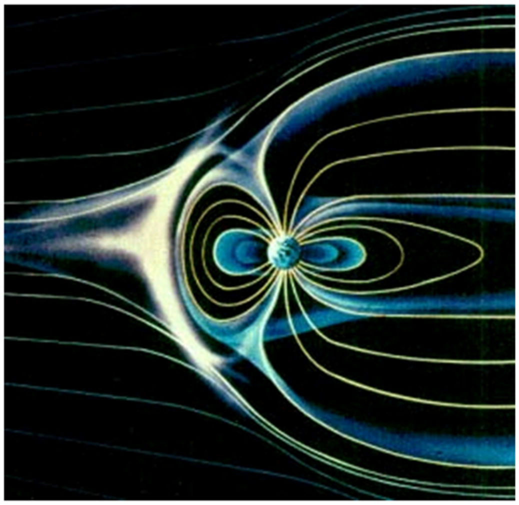
Heat from the sun = **Solar Wind**

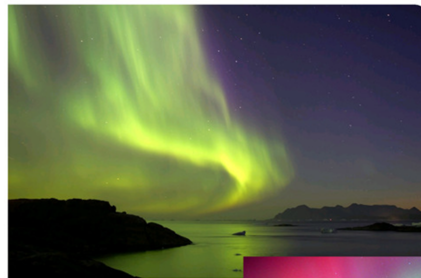
The solar wind contains roughly equal number of electrons and protons, along with a few heavier ions, and blows continuously from the surface of the Sun at an average velocity of about 400 km/second. This is a remarkable velocity: particles in the solar wind from the Sun's surface travel at a speed that would allow them to go from Knoxville to Memphis in less than 2 seconds! This wind leads to a mass loss of more than 1 million tons of material per second, which may seem like a large number, but is insignificant relative to the total mass of the Sun.

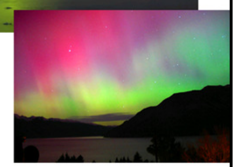



Info from: http://csep10.phys.utk.edu/astr162/lect/sun/wind.html

# The Earth Has a Magnetic Field which protects the Earth's Atmosphere



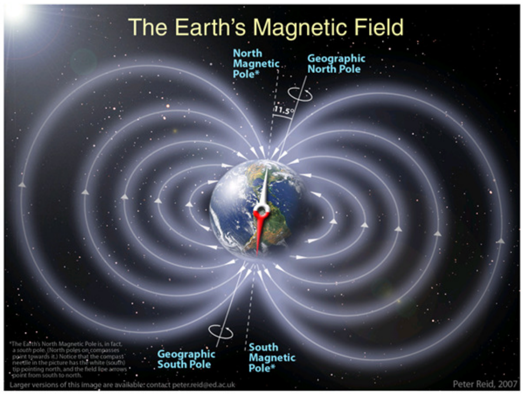




**The Poles of the magnetic field at the North and South create the effects of the Northern and Southern Lights during Solar Flares**

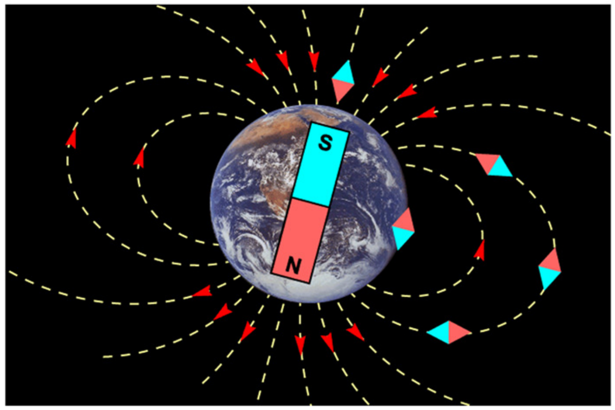
# But What Creates The Magnetic Field

**The Earth**




**A Common Bar Magnet**

So, Is there a Giant Bar Magnet inside the Earth?... (suspense…)




End suspense. NO! Then what is inside the earth that creates the same field?

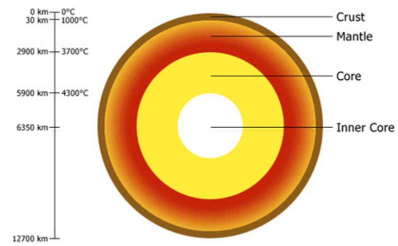
# Composition of Early Earth

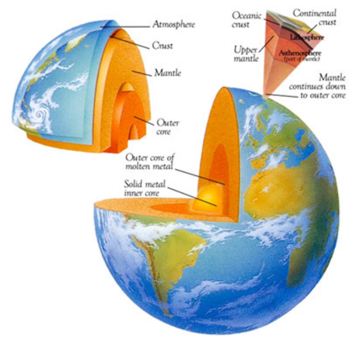
<u>Inner core:</u>

It is 3,200 - 3,960 miles (5,150-6,370 km) below the earth's surface and mainly consists of iron, nickel and some lighter elements (probably sulphur, carbon, oxygen, silicon and potassium. The temperature in the inner core is about 9032 - 10832 ºF (5000-6000 ºC). Because of the high pressure, the core is solid. The average density of the core is about 15g/cm³.

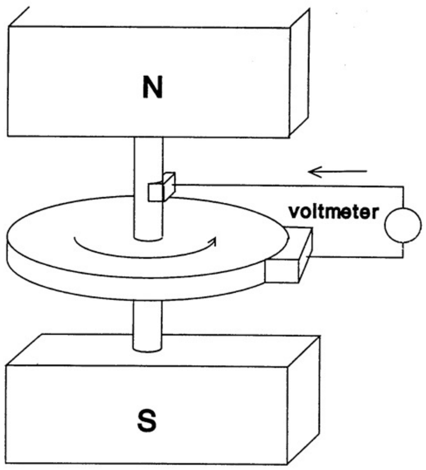
<u>Outer core:</u>

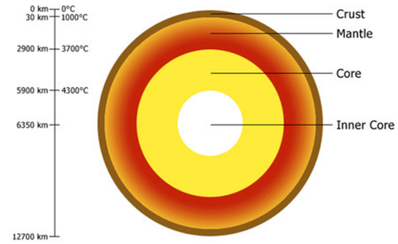
The outer core is at 1,800 - 3,200 miles (2,890-5,150 km) below the earth's surface. The outer core is liquid and mainly consists of iron, some nickel and about 10% sulphur and oxygen. The temperature in the outer core is about 7200 - 9032 ºF (4000-5000ºC). The density of the outer core is between the 10g/cm³ and 12.3g/cm³. The outer core and inner core together cause the earth's magnetism.



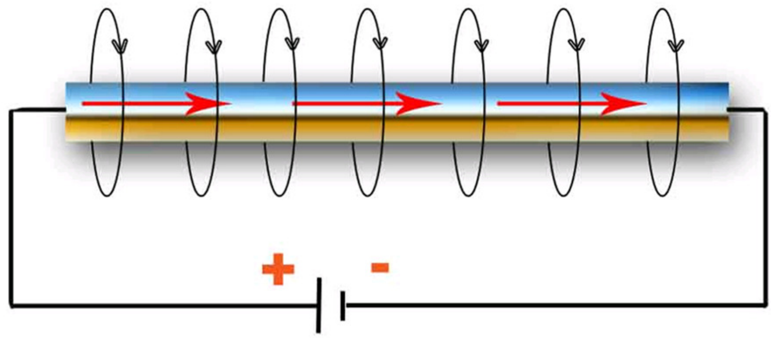



# Dynamo Theory



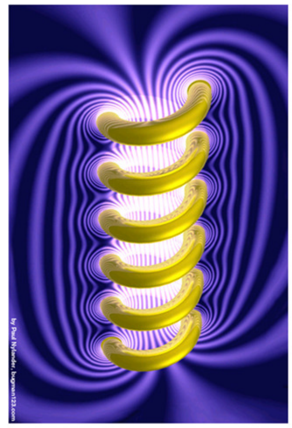

# An ELECTRIC Current creates a MAGNETIC Field



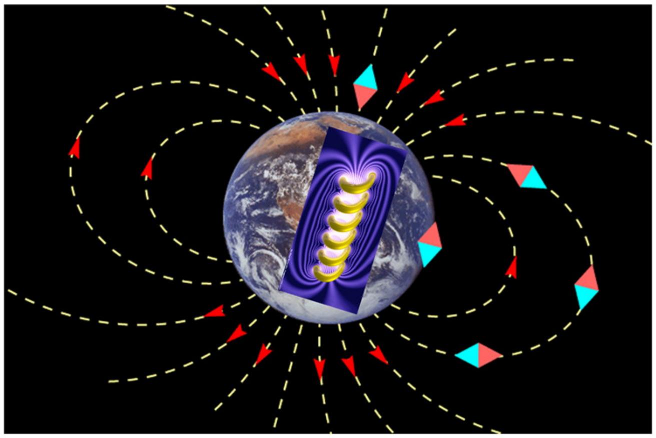

**Battery creating a current in a wire which creates a magnetic field around the wire**

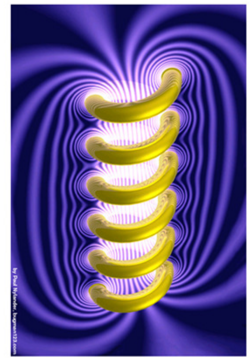
## This is because the Molten Iron creates a current in a loop around the earth, actually, many loops

- This is called, **a solenoid**.
- Let's Test that the theory works.
  - To model the solid inner core of the earth, a metal screw
  - To model the molten outer core creating an electric current around the inner core, wire with electricity from a battery going through it



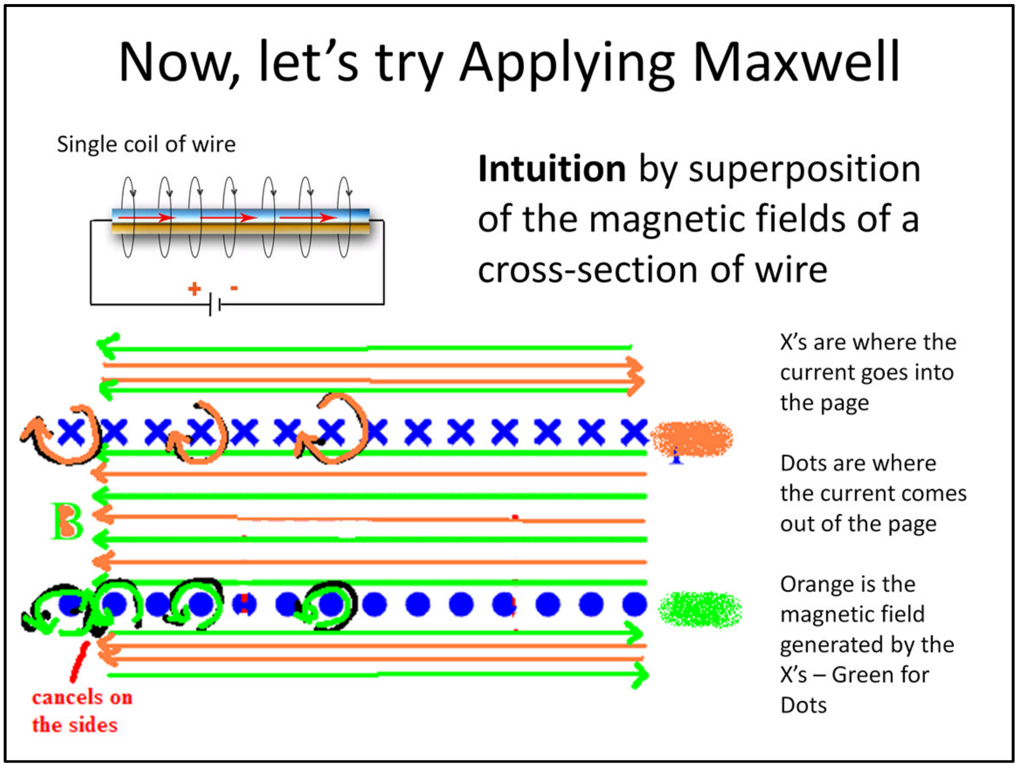
# Yes! It works! Thus:





THE EARTH'S MAGNETIC FIELD IS CREATED BY A GIANT NATURAL SOLENOID!!!

Notice the EM theory holds, the field created is like composing the magnetic fields of a bunch of loops of wire. ☺



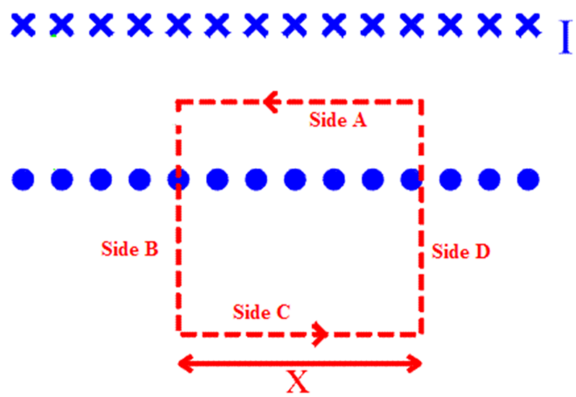

Use symmetry and cancellation to show that the magnetic field is zero everywhere but the center. (zero on the outside as long as the solenoid is very long relative to how wide it is) But how strong is it in the center? You can figure this out using explicit application of ampere's law – the math technique is a bit tricky however

# Explicit Mathematics

Use Ampere's Law to, given the Electric Flux through the below Loops, Compute the Magnetic Field. (Notice, Electric Field is just Current x number of coils within the loop.

$$\oint \mathbf{B} \cdot dl = \mathbf{I} + \varepsilon \frac{d}{dt} \iint \mathbf{E} \cdot ds$$

Ampere's Law

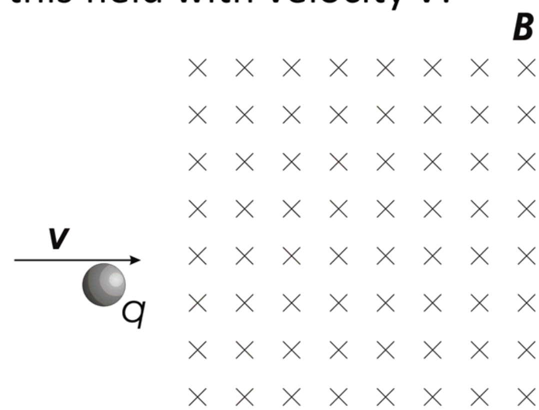



The mechanical use of Ampere's law in this is actually fairly complicated. When you take the integral around the loop (left part of the equation) you actually only want to count up the components** of the magnetic field vectors that are parallel to the direction of the loop at those points. (aka, a magnetic field at magnitude M iat a 45 degree angle to the direction of the loop at some point, only counts as M/2 magnetrons going along the line – because of the angle it is only 'half-going' along the line). Also note that in all cases when you take the integral over a surface, you only want the components of the electric field vectors that are perpendicular to the surface. (one going through at a 45 degree angle is only 'half-going-through' ☺)

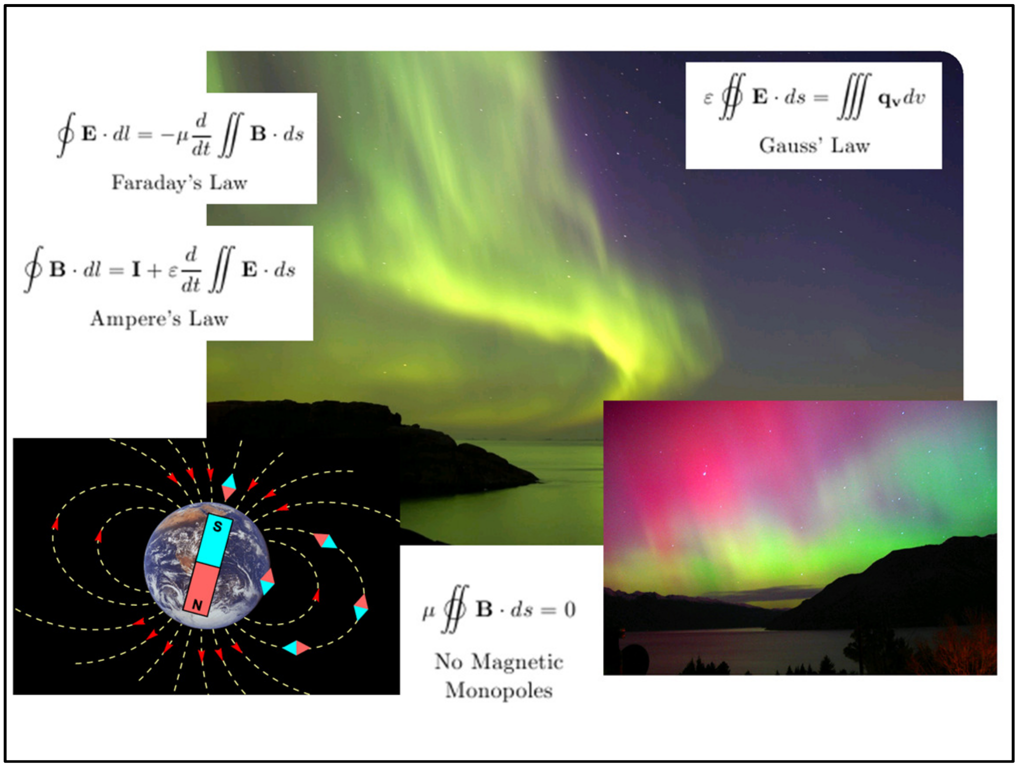
** the generalized component of a vector in one direction is defined here:
http://id.mind.net/~zona/mstm/physics/mechanics/vectors/findingComponents/findingComponents.htm
(first hit that looked good on google)

# Another Cool Problem

- Consider a uniform magnetic field created by a really big solenoid. Say it goes into this page. What happens to an electron when it is shot into this field with velocity v?




(remember, applying a force always perpendicular to velocity is like what you do when you swing a weight on a string around your head like a lasso)


$\oint \mathbf{E} \cdot dl = -\mu \frac{d}{dt} \iint \mathbf{B} \cdot ds$
Faraday's Law
$\oint \mathbf{B} \cdot dl = \mathbf{I} + \varepsilon \frac{d}{dt} \iint \mathbf{E} \cdot ds$
Ampere's Law
$\varepsilon \oiint \mathbf{E} \cdot ds = \iiint \mathbf{q_v} dv$
Gauss' Law
S
N
$\mu \oiint \mathbf{B} \cdot ds = 0$
No Magnetic Monopoles